



“Tell me that you have found no sign of  
New Physics again, I dare you.  
I double dare you. Tell me  
one more goddamn **time!**”

# Electroweak Precision Observables (at Future Colliders)

*Sven Heinemeyer, IFT/IFCA (CSIC, Madrid/Santander)*

Madrid, 06/2019

1. Motivation
2. Electroweak Precision Observables (EWPOs)
3. EWPO status
4. EWPO future
5. SM Higgs
6. BSM Higgs
7. Conclusions

# 1. Introduction

## Experimental situation:

(HL-)LHC/ILC/CLIC/FCC-ee/CEPC/...  
will provide (high!) accuracy **measurements!**

## Theory situation:

- Measurements are performed using **theory predictions**
- **measured observables** have to be compared with **theoretical predictions**  
(in various models: SM, MSSM, ...)

Full uncertainty is given by the (linear) sum of  
experimental and theoretical uncertainties!

⇒ Experimental precision can only fully be exploited  
with theory uncertainties at the same level of accuracy!

Many results shown here based on:

[arXiv:1906.05379]

Write-up for FCC-ee physics WG2 – Precision EW Calculations

## Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee

A. Freitas<sup>\*1</sup>, S. Heinemeyer<sup>\*2</sup>, M. Beneke<sup>3</sup>, A. Blondel<sup>4</sup>, S. Dittmaier<sup>5</sup>,  
J. Gluza<sup>6,7</sup>, A. Hoang<sup>8</sup>, S. Jadach<sup>9</sup>, P. Janot<sup>10</sup>, J. Reuter<sup>11</sup>, T. Riemann<sup>6,12</sup>,  
C. Schwinn<sup>13</sup>, M. Skrzypek<sup>8</sup>, and S. Weinzierl<sup>14</sup>

⇒ Here: focus on  $e^+e^-$  precision

⇒ should be taken into account by “exp groups”!

⇒ Here: current status and future of EWPO/Higgs TH calculations  
what may be achievable in TH calc. in  $\mathcal{O}(20)$  years

## Where we need theory prediction:

### 1. Prediction of the measured quantity

Example:  $M_W$ ,  $\Gamma(H \rightarrow b\bar{b})$

→ at the same level or better as the experimental precision

### 2. Prediction of the measured process to extract the quantity

Example:  $e^+e^- \rightarrow W^+W^-$ ,  $e^+e^- \rightarrow ZH$

→ better than then “pure” experimental precision

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## Two types of theory uncertainties:

### 1. intrinsic: missing higher orders

### 2. parametric: uncertainty due to exp. uncertainty in SM input parameters

Example:  $m_t$ ,  $m_b$ ,  $\alpha_s$ ,  $\Delta\alpha_{\text{had}}$ ,  $\dots$

## Options for the evaluation of intrinsic uncertainties:

1. Determine all **prefactors** of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the **loop is  $\mathcal{O}(1)$**
2. Take the known contribution at  $n$ -loop and  $(n - 1)$ -loop and thus estimate the  $n + 1$ -loop contribution:

$$\frac{(n + 1)(\text{estimated})}{n(\text{known})} \approx \frac{n(\text{known})}{(n - 1)(\text{known})}$$

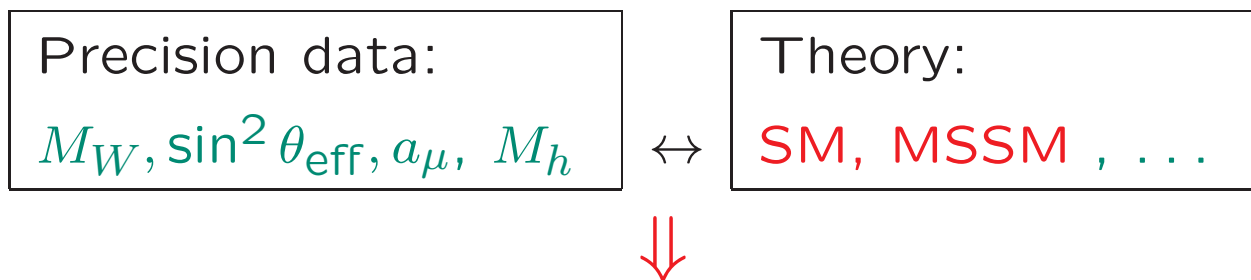
$\Rightarrow$  simplified example! Has to be done  
“coupling constant by coupling constant”

3. Variation of  $\mu^{\overline{\text{MS}}}$  (QCD!, EW?)
4. Compare **different renormalizations**

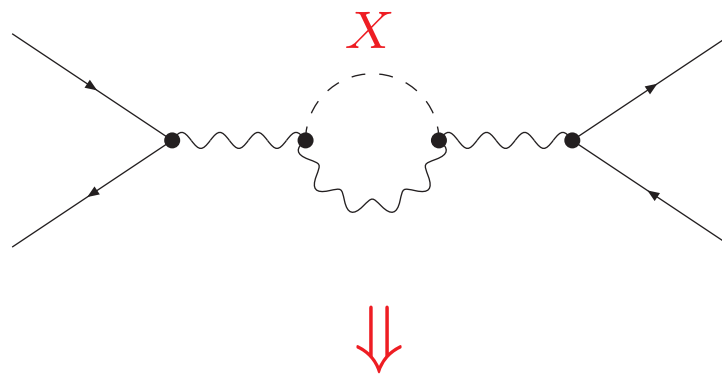
$\Rightarrow$  Mostly used here: 1 & 2

## 2. Electroweak Precision Observables

Comparison of observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections, e.g.  $X$



SM: limits on  $M_H$ , BSM: limits on  $M_X$

Very high accuracy of measurements and theoretical predictions needed  
 $\Rightarrow$  only models “ready” so far: SM, MSSM



## Precision observables in the SM and the MSSM

$M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $M_h$ ,  $(g-2)_\mu$ ,  $b$  physics, ...

A) Theoretical prediction for  $M_W$  in terms

of  $M_Z, \alpha, G_\mu, \Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

Evaluate  $\Delta r$  from  $\mu$  decay  $\Rightarrow M_W$

One-loop result for  $M_W$  in the SM:

[A. Sirlin '80] , [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{1\text{-loop}} &= \Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho + \Delta r_{\text{rem}}(M_H) \\ &\sim \log \frac{M_Z}{m_f} \quad \sim m_t^2 \quad \log(M_H/M_W) \\ &\sim 6\% \quad \sim 3.3\% \quad \sim 1\% \end{aligned}$$

## Precision observables in the SM and the MSSM

$M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $M_h$ ,  $(g-2)_\mu$ ,  $b$  physics, ...

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$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

**B)** Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left( 1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

## Corrections to $M_W$ , $\sin^2 \theta_{\text{eff}}$ $\rightarrow$ approximation via the $\rho$ -parameter:

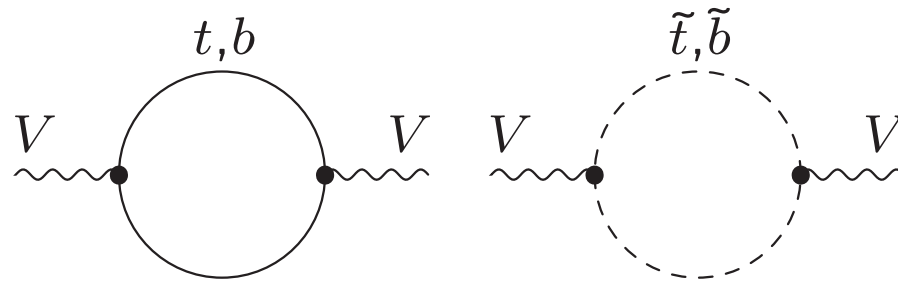
$\rho$  measures the relative strength between  
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \quad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$  gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho, \quad \Delta \sin^2 \theta_W^{\text{eff}} \approx - \frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho$$



$$\Delta\rho^{\text{SUSY}} \text{ from } \tilde{t}/\tilde{b} \text{ loops} > 0 \quad \Rightarrow \quad M_W^{\text{SUSY}} \gtrsim M_W^{\text{SM}}$$

## All the EWPO:

$$M_W \quad (\text{best from threshold scan})$$

$$\sigma_{\text{had}}^0 = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \rightarrow f\bar{f}], \quad (\text{from a fit to } \sigma_f(s) \text{ at various values of } s)$$

$$R_\ell = [\sum_q \sigma_q(M_Z^2)] / \sigma_\ell(M_Z^2), \quad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2) / [\sum_q \sigma_q(M_Z^2)], \quad (q = b, c)$$

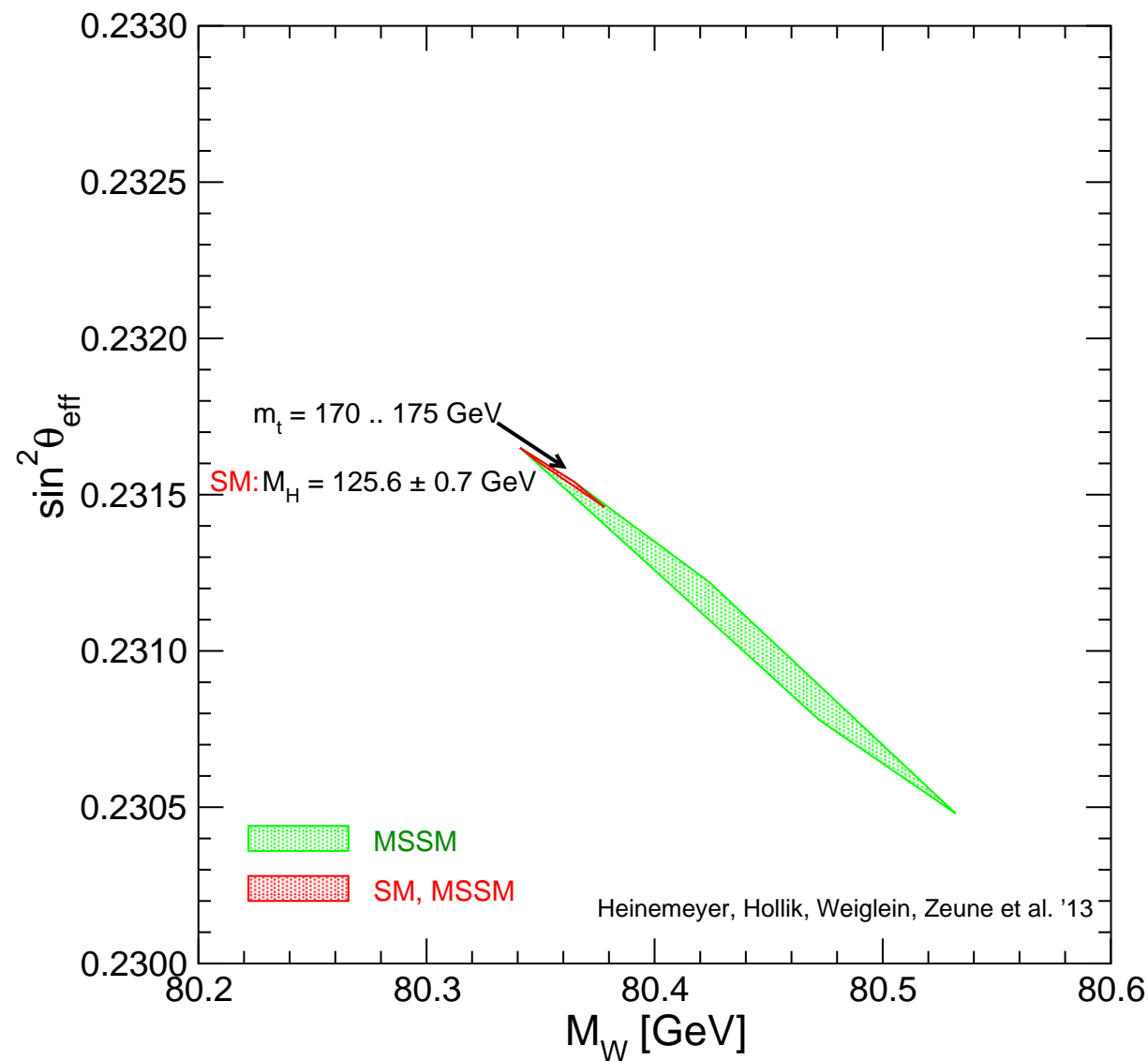
$$A_{\text{FB}}^f = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f,$$

$$A_{\text{LR}}^f = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e |P_e|$$

$$\mathcal{A}_f = 2 \frac{g_{V_f}/g_{A_f}}{1 + (g_{V_f}/g_{A_f})^2} = \frac{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f}{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f + 8(|Q_f| \sin^2 \theta_{\text{eff}}^f)^2} \quad (f = \ell, b, \dots)$$

# What $M_W$ and $\sin^2 \theta_{\text{eff}}$ precision do we want?

[S.H., W. Hollik, G. Weiglein, L. Zeune et al. '13]



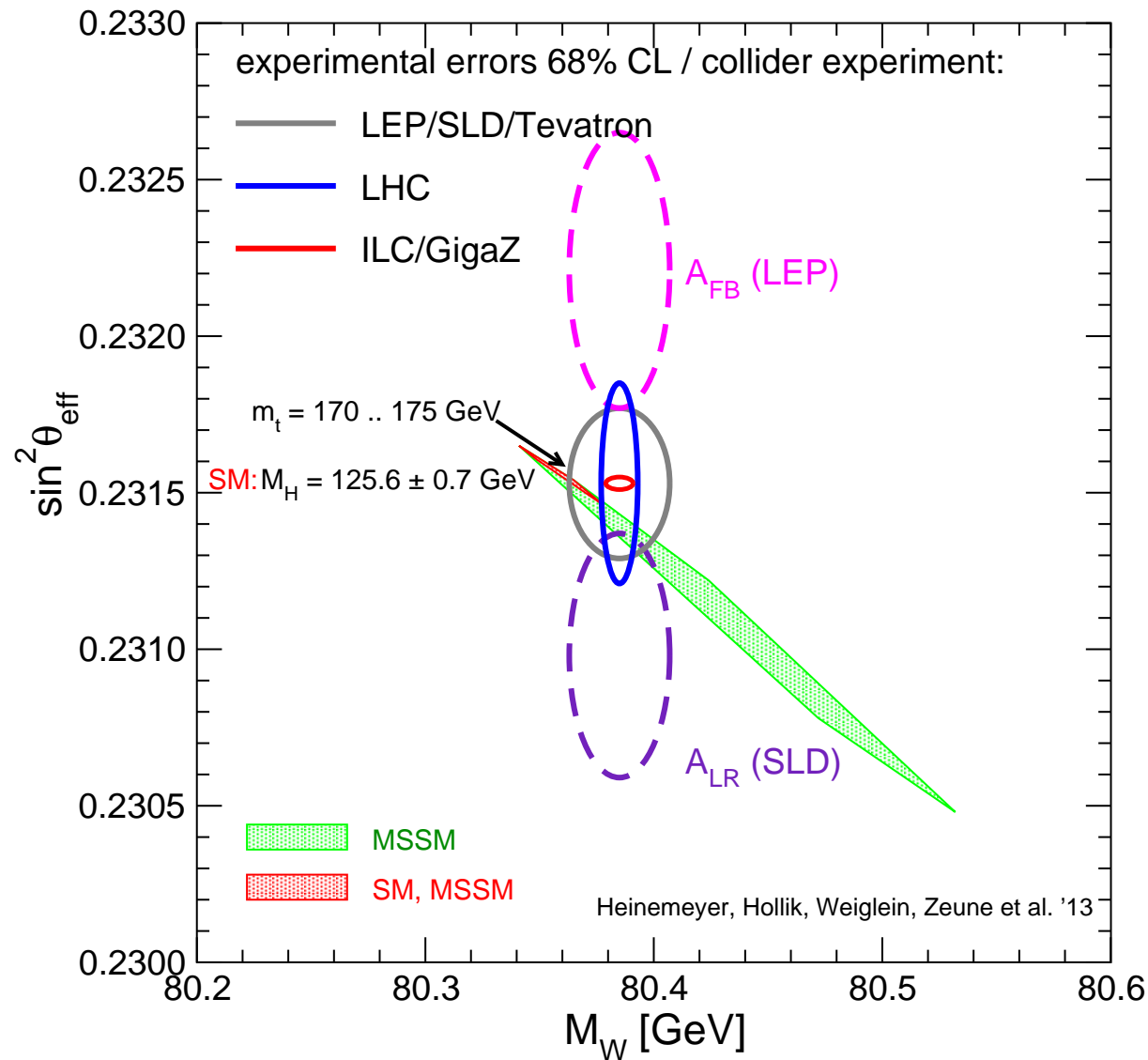
MSSM band:  
scan over  
SUSY masses

overlap:  
SM is MSSM-like  
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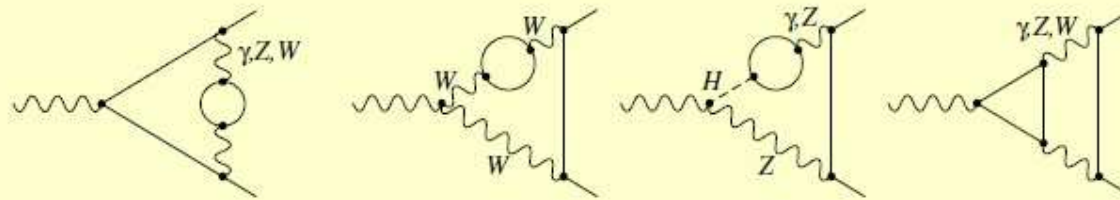
SM band:  
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### 3. EWPO Status

#### Existing higher-order corrections to the EWPO

[taken from A. Freitas '16]

Known corrections to  $\Delta r$ ,  $\sin^2 \theta_{\text{eff}}^f$ ,  $g_V f$ ,  $g_A f$ :



- Complete NNLO corrections ( $\Delta r$ ,  $\sin^2 \theta_{\text{eff}}^f$ ) Freitas, Hollik, Walter, Weiglein '00  
Awramik, Czakon '02; Onishchenko, Veretin '02  
Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06  
Hollik, Meier, Uccirati '05,07; Degrandi, Gambino, Giardino '14
- “Fermionic” NNLO corrections ( $g_V f$ ,  $g_A f$ ) Czarnecki, Kühn '96  
Harlander, Seidensticker, Steinhauser '98  
Freitas '13,14
- Partial 3/4-loop corrections to  $\rho/T$ -parameter  
 $\mathcal{O}(\alpha_t \alpha_s^2)$ ,  $\mathcal{O}(\alpha_t^2 \alpha_s)$ ,  $\mathcal{O}(\alpha_t \alpha_s^3)$  Chetyrkin, Kühn, Steinhauser '95  
Faisst, Kühn, Seidensticker, Veretin '03  
Boghezal, Tausk, v. d. Bij '05  
Schröder, Steinhauser '05; Chetyrkin et al. '06  
Boghezal, Czakon '06

$$(\alpha_t \equiv \frac{y_t^2}{4\pi})$$

## Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.
$M_W$ [MeV]	15	4 ( $\alpha^3, \alpha^2\alpha_s$ )
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	16	4.5 ( $\alpha^3, \alpha^2\alpha_s$ )
$\Gamma_Z$ [MeV]	2.3	0.5 ( $\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$ )
$R_b$ [ $10^{-5}$ ]	66	15 ( $\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$ )
$R_l$ [ $10^{-3}$ ]	25	5 ( $\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$ )



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## Parametric uncertainties:

Quantity	$\delta m_t = 0.9 \text{ GeV}$	$\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$	$\delta M_Z = 2.1 \text{ MeV}$
$\delta M_W^{\text{para}}$ [MeV]	5.5	2	2.5
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⇒ Current intrinsic/parametric uncertainties are substantially smaller than current experimental uncertainties :-)

## Intrinsic uncertainties:

NEW:  $\alpha_{\text{bos}}^2$  calc. [Dubovyka et al. '18]

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## Additional uncertainty for $M_W$ from threshold scan:

Not only  $e^+e^- \rightarrow W^{(*)}W^{(*)}$ , but  $e^+e^- \rightarrow WW \rightarrow 4f$  needed

### Current status:

full one-loop for  $2 \rightarrow 4$  process

[A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02]

$\Rightarrow$  extraction of  $M_W$  at the level of  $\sim 6$  MeV

### Most recent improvement:

leading 2L corrections from EFT

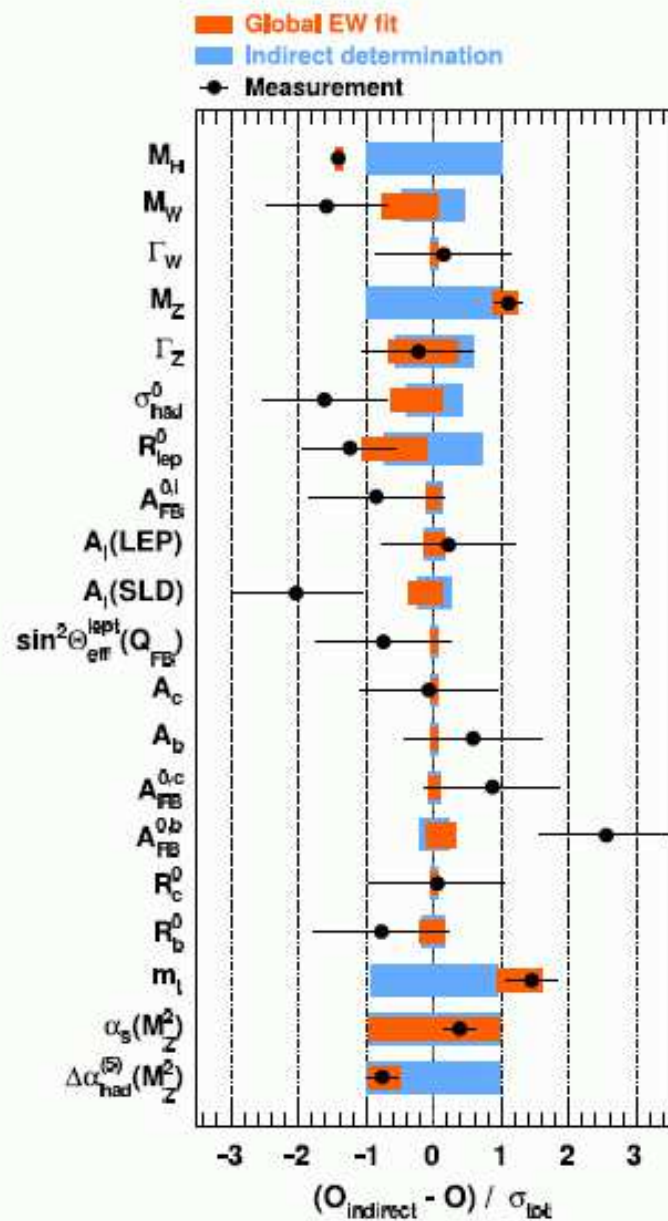
[Actis, Beneke, Falgari, Schwinn '08]

$\Rightarrow$  impact on  $M_W$  at the level of  $\sim 3$  MeV

$\Rightarrow$  well under control for LEP data

## Overview about all EWPO:

[taken from A. Freitas '16]

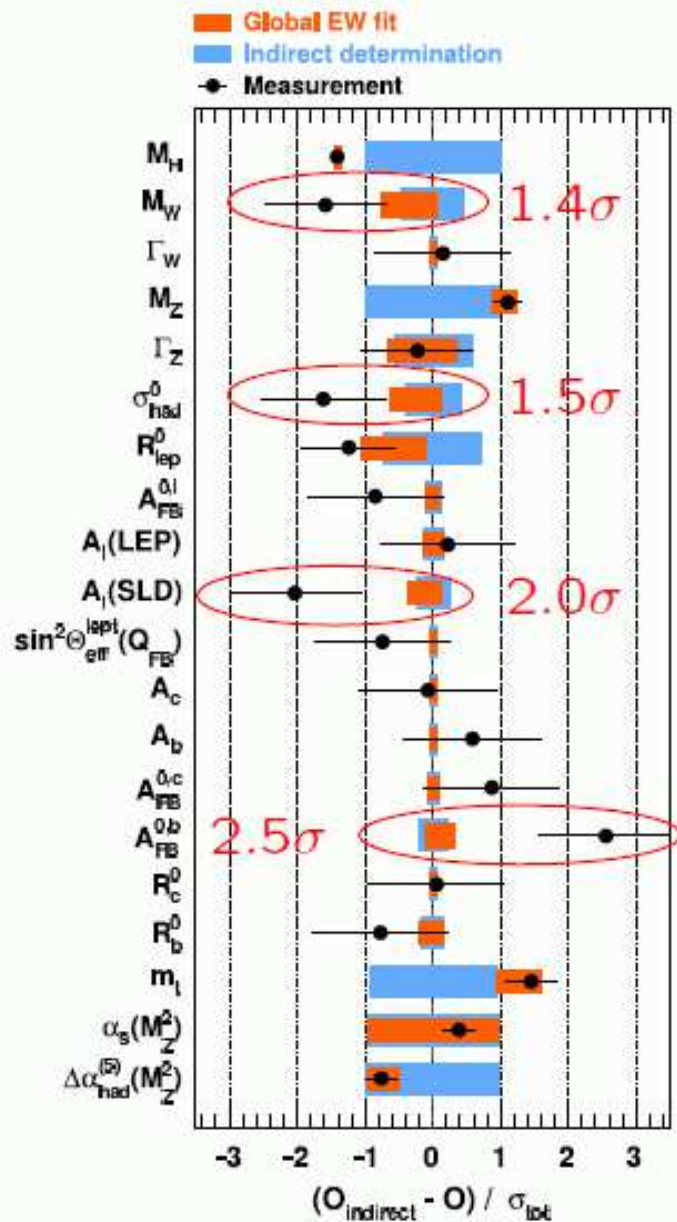


Surprisingly good agreement:

$$\chi^2/\text{d.o.f.} = 18.1/14 \quad (p = 20\%)$$

Most quantities measured with  
1%–0.1% precision

GFitter coll. '14



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A few interesting deviations:

$$M_W \quad (\sim 1.4\sigma)$$

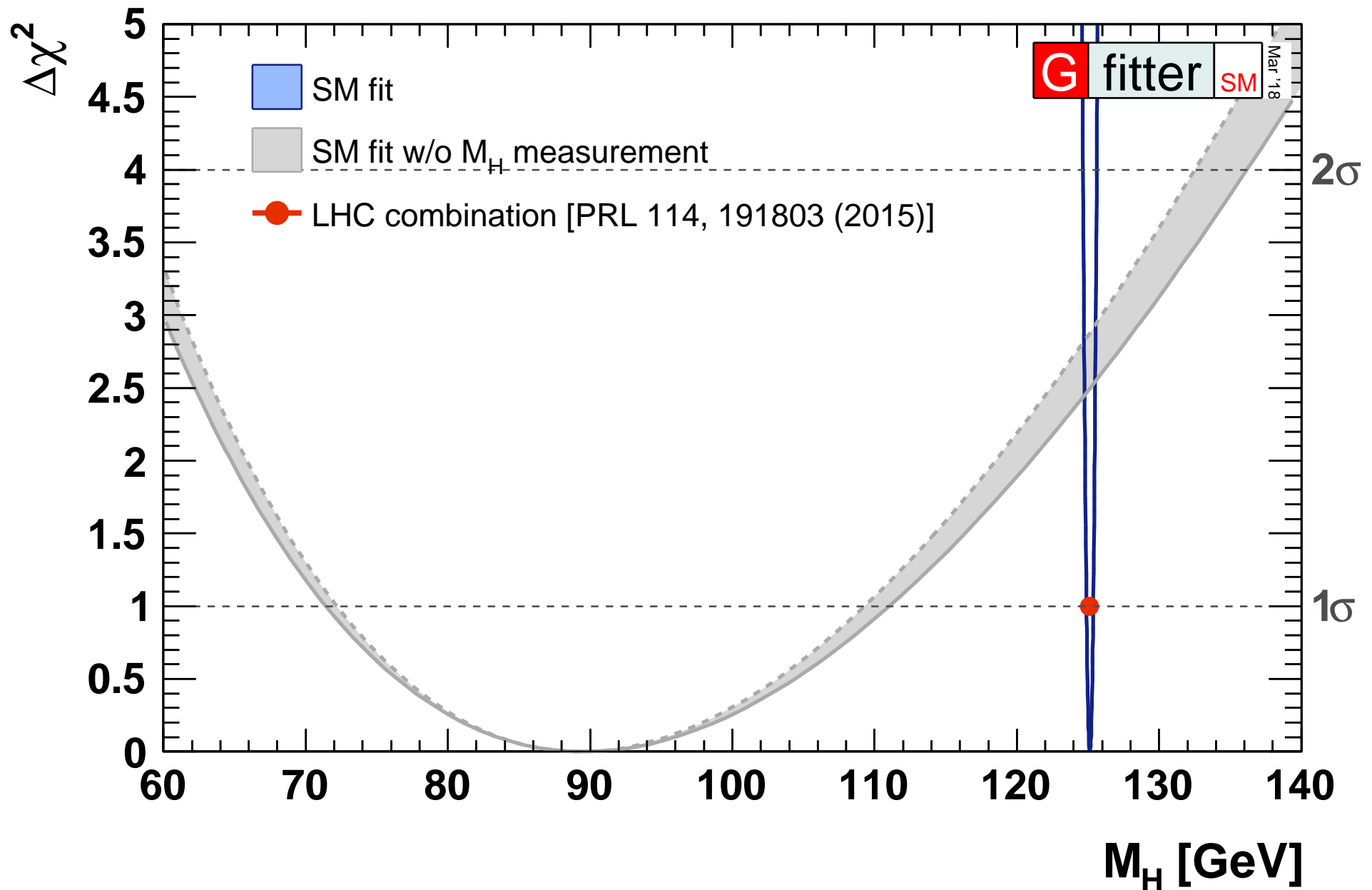
$$\sigma_{\text{had}}^0 \quad (\sim 1.5\sigma)$$

$$A_{\ell}(\text{SLD}) \quad (\sim 2\sigma)$$

$$A_{\text{FB}}^b \quad (\sim 2.5\sigma)$$

$$(g_{\mu} - 2) \quad (\sim 3\sigma)$$

GFitter coll. '14



## 4. EWPO Future

### Our future estimates:

- assume to go substantially beyond what is known now
- assume that many theorists will put many<sup>2</sup> hours of work into it (motivation?)
- do not assume that magically new calculational methods are invented
- are overall optimistic

⇒ they should be taken seriously!

⇒ An honest evaluation of theory uncertainties will increase the robustness of a future collider physics case!



## What is needed to match the FC precision?

### Compare:

1. FC (pure) **experimental** (anticipated) precision
2. **Intrinsic** uncertainties
3. **Parametric** uncertainties  
→ taking into account the improved precision of SM parameters at the FC

### Combined uncertainty:

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Intrinsic uncertainties:  $\Rightarrow$  can be the limiting factor!

Quantity	ILC	FCC-ee	Current intrinsic unc.	Projected unc.
$M_W$ [MeV]	3	0.5	4 ( $\alpha^3, \alpha^2\alpha_s$ )	1
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	1.3	0.6	4.5 ( $\alpha^3, \alpha^2\alpha_s$ )	1.5
$\Gamma_Z$ [MeV]	1	0.1	0.5 ( $\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$ )	0.2 (?)
$R_b$ [ $10^{-5}$ ]	15	6	15 ( $\alpha^3, \alpha^2\alpha_s$ )	7 (?)
$R_l$ [ $10^{-3}$ ]	10??	1	5 ( $\alpha^3, \alpha^2\alpha_s$ )	1.5 (?)

These calculations are required for the projection:

- complete  $\mathcal{O}(\alpha\alpha_s^2)$  corrections
- fermionic  $\mathcal{O}(\alpha^2\alpha_s)$  corrections
- double-fermionic  $\mathcal{O}(\alpha^3)$  corrections
- leading four-loop corrections enhanced by the top Yukawa coupling
- the  $\mathcal{O}(\alpha_{\text{bos}}^2)$  corrections are done now [Dubovyka et al. '18]

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

## Parametric uncertainties:

1.  $M_H$ : better than 50 MeV  $\Rightarrow$  negligible
2.  $M_Z$ :  $\sim 0.1$  MeV with negligible theory uncertainties  $\Rightarrow$  negligible
3.  $\alpha_s(M_Z)$ : from (mainly)  $R_\ell$   
 $\delta\alpha_s^{\text{exp}} \sim 10^{-4}$ ,  $\delta\alpha_s^{\text{theo}} \sim 1.5 \times 10^{-4}$
4.  $m_t$ : from threshold scan  
 $\delta m_t^{\text{exp}} \sim \mathcal{O}(10 \text{ MeV})$   
 $\delta m_t^{\text{theo}} \sim 50 \text{ MeV}$  (NNNLO/NNLL  $\oplus$   $1S \rightarrow \overline{MS}$   $\oplus$   $\delta\alpha_s$ )
5.  $m_b$ : from lattice calculations  $\Rightarrow$  negligible for EWPO  
 $\delta m_b \sim 10 \text{ MeV}$  (still under discussion, too optimistic?)
6.  $\Delta\alpha_{\text{had}}$ : BES III and Belle II:  $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$   
better from measurements “around the  $Z$  pole?”

## Uncertainty budget for $m_t$ :

[talk by A. Hoang '15]

Msbar mass error budget (from threshold scan)

$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\text{conversion}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

$$\delta\alpha_s(M_z) = 0.001$$

⇒ improvement in  $\alpha_s$  crucial

$e^+e^-$  collider: precision measurement:

$$R_l := \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow l^+l^-)}$$

Improvement down to  $\delta^{\text{exp}}\alpha_s \sim 0.001 - 0.0001$  possible?!

Note: **TH uncertainty** (assuming fermionic 3-loop corrections):

$$\delta R_l^{\text{theo}} \sim 0.0015 \Rightarrow \delta\alpha_s^{\text{theo}} \sim 0.00015$$

⇒ hard to beat ...

## $M_W$ parametric:

parametric today:  $\delta m_t = 0.9$  GeV,  $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$ ,  $\delta M_Z = 2.1$  MeV

$$\delta M_W^{\text{para}, m_t} = 5.5 \text{ MeV}, \quad \delta M_W^{\text{para}, \Delta\alpha_{\text{had}}} = 2 \text{ MeV}, \quad \delta M_W^{\text{para}, M_Z} = 2.5 \text{ MeV}$$

parametric future:  $\delta m_t^{\text{fut}} = 0.05$  GeV,  $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$ ,  $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1$  MeV

$$\Delta M_W^{\text{para}, \text{fut}, m_t} = 0.5 \text{ MeV}, \quad \Delta M_W^{\text{para}, \text{fut}, \Delta\alpha_{\text{had}}} = 1 \text{ MeV}, \quad \Delta M_W^{\text{para}, \text{fut}, M_Z} = 0.2/0.02 \text{ MeV}$$

## $\sin^2 \theta_{\text{eff}}$ parametric: $[10^{-5}]$

parametric today:  $\delta m_t = 0.9$  GeV,  $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$ ,  $\delta M_Z = 2.1$  MeV

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}, m_t} = 3.0, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para}, \Delta\alpha_{\text{had}}} = 3.6, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para}, M_Z} = 1.4$$

parametric future:  $\delta m_t^{\text{fut}} = 0.05$  GeV,  $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$ ,  $\delta M_Z^{\text{ILC/FCC-ee}} = 1/0.1$  MeV

$$\Delta \sin^2 \theta_{\text{eff}}^{\text{para}, \text{fut}, m_t} = 0.2, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para}, \text{fut}, \Delta\alpha_{\text{had}}} = 1.8, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para}, \text{fut}, M_Z} = 0.65/0.07$$

## Additional uncertainty for $M_W$ from threshold scan:

Not only  $e^+e^- \rightarrow W^{(*)}W^{(*)}$ , but  $e^+e^- \rightarrow WW \rightarrow 4f$  needed

### Current status:

full one-loop for  $2 \rightarrow 4$  process

[A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02]

$\Rightarrow$  extraction of  $M_W$  at the level of  $\sim 6$  MeV

### Most recent improvement:

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[Actis, Beneke, Falgari, Schwinn '08]

$\Rightarrow$  impact on  $M_W$  at the level of  $\sim 3$  MeV

$\Rightarrow$  full 2L for  $2 \rightarrow 4$  process not foreseeable

### Potentially possible:

2L resummed higher-order terms for  $e^+e^- \rightarrow WW$  and  $W \rightarrow ff'$

$\Rightarrow$  extraction of  $M_W$  at  $\sim 1$  MeV??  $\oplus$  pure exp. uncertainty of  $\sim 3/0.5$  MeV

## Summary of future parametric uncertainties:

Quantity	ILC	FCC-ee	future parametric unc.	Main source
$M_W$ [MeV]	$3 \oplus 1$	$0.5 \oplus 1$	1	$\delta(\Delta\alpha_{\text{had}})$
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	1.3	0.6	2	$\delta(\Delta\alpha_{\text{had}})$
$\Gamma_Z$ [MeV]	1	0.1	0.5	
$R_b$ [ $10^{-5}$ ]	15	6	$< 1$	$\delta\alpha_s$

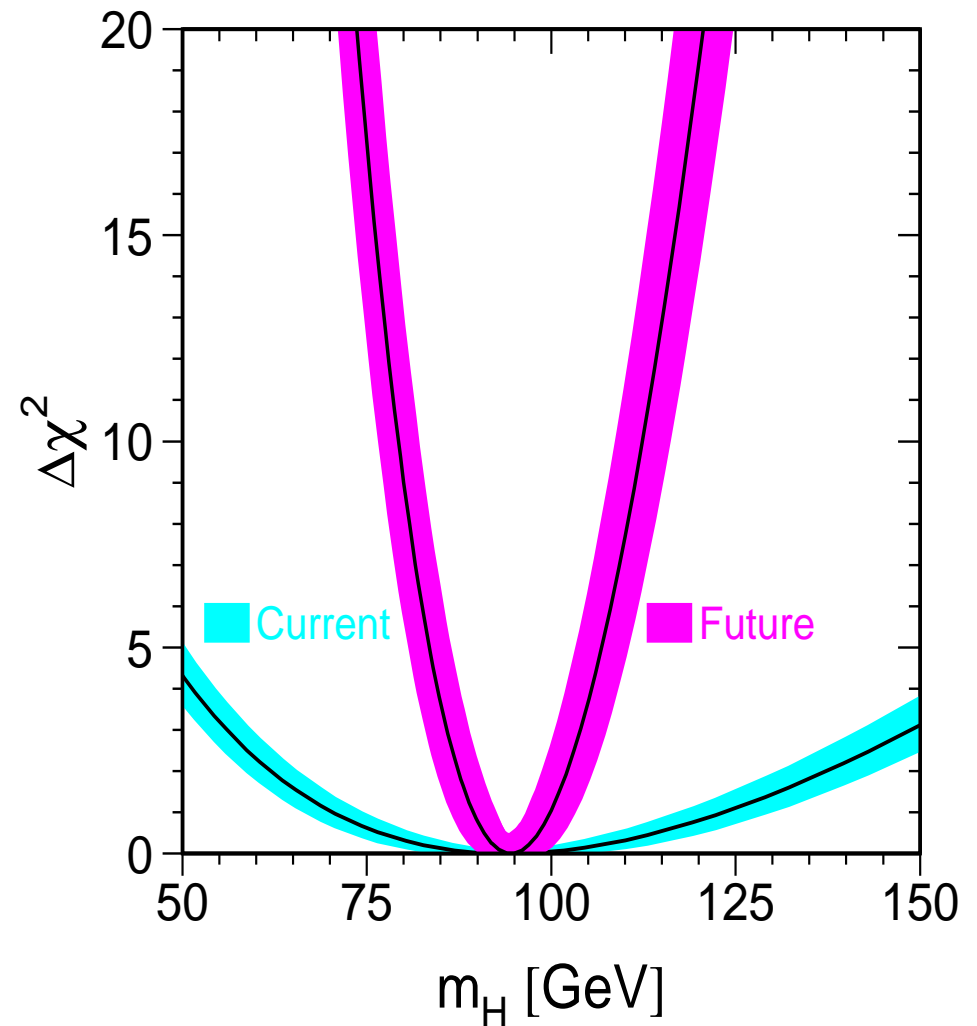
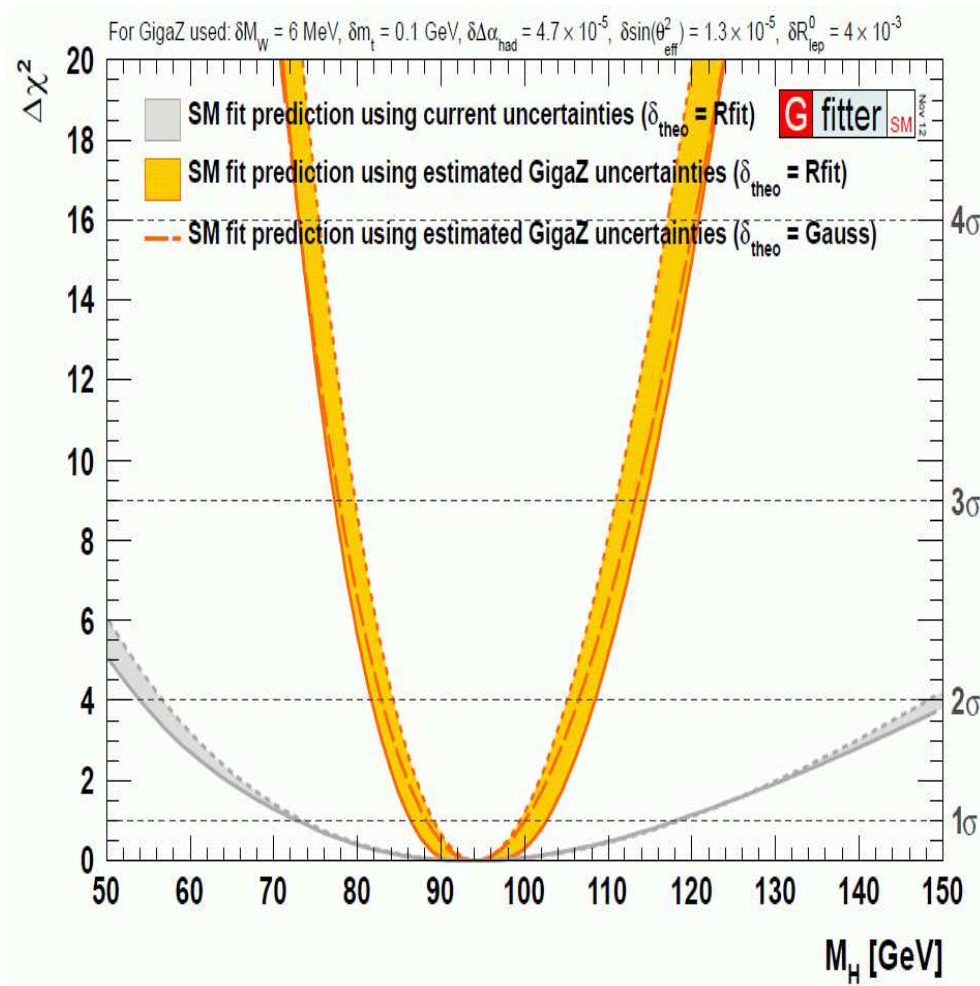
⇒ add quadratic to experimental uncertainties!

⇒ add linearly to intrinsic uncertainties!

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

# Precise $M_H$ test with the ILC precision:

[GFitter '13] [LEPEWWG '13]



$$\Rightarrow \delta M_H^{\text{ind}} \lesssim 6 \text{ GeV}$$

$\Rightarrow$  extremely sensitive test of SM (and BSM) possible

$\Leftarrow$  to be redone incl. all TH unc.



## One more word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

We know that BSM physics must exist! (DM, gravity, ...)

As soon as BSM physics will be discovered, an evaluation of the EWPO in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

## 5. SM Higgs (the easy case)

Initial measurement:  $\sigma \times \text{BR}$

recoil method:  $e^+e^- \rightarrow ZH, Z \rightarrow e^+e^-, \mu^+\mu^-$

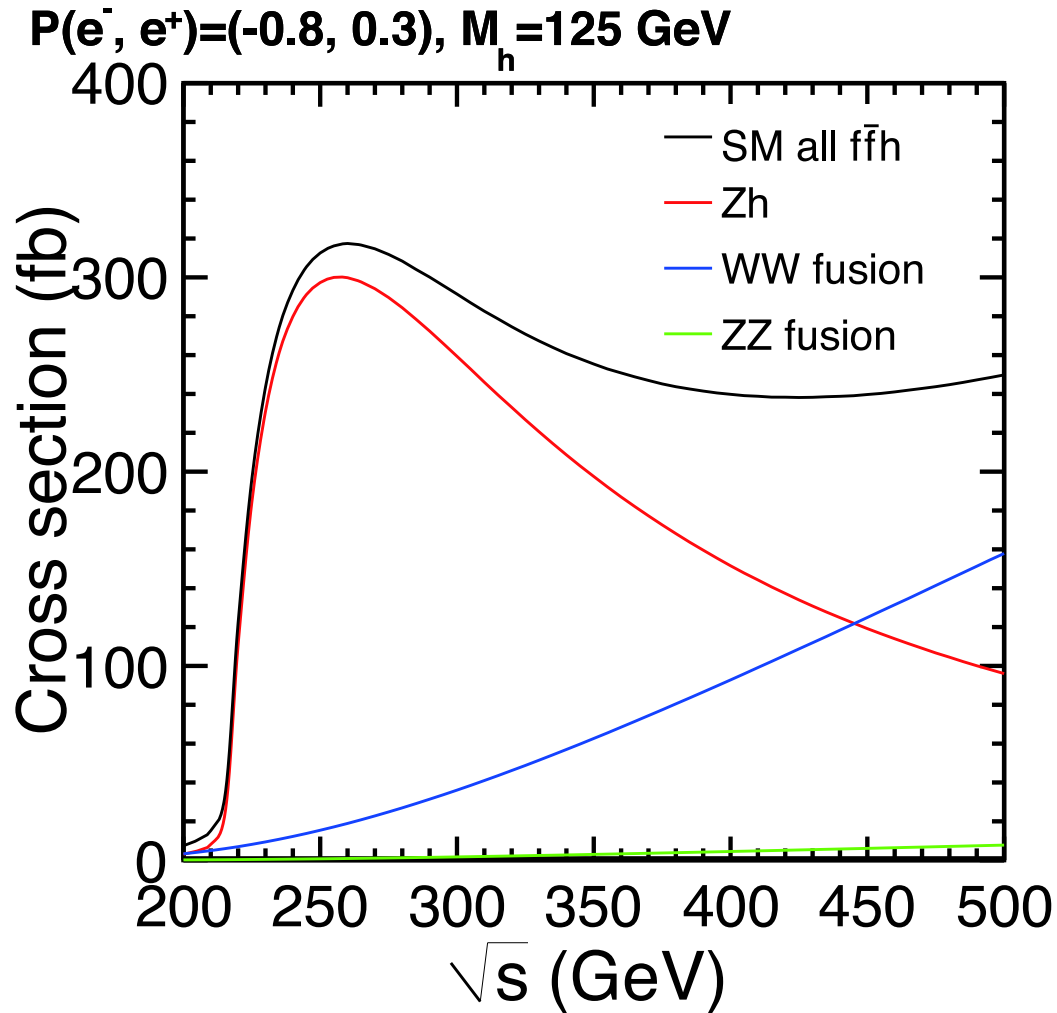
$\Rightarrow$  measurement of the Higgs production cross section

$\Rightarrow$  NO additional theoretical assumptions needed for absolute determination of partial widths

$\Rightarrow$  indirect measurement of total width

$\Rightarrow$  direct extraction of partial widths (couplings)

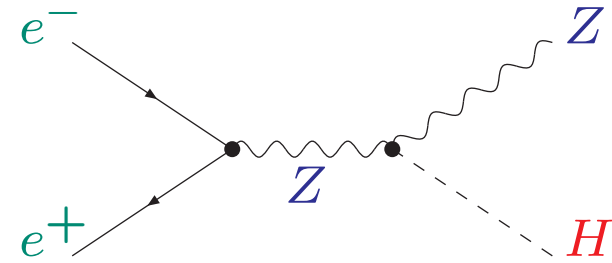
## Higgs production cross sections:



$\sqrt{s} \sim 250 \text{ GeV}$ , Higgs-strahlung dominated

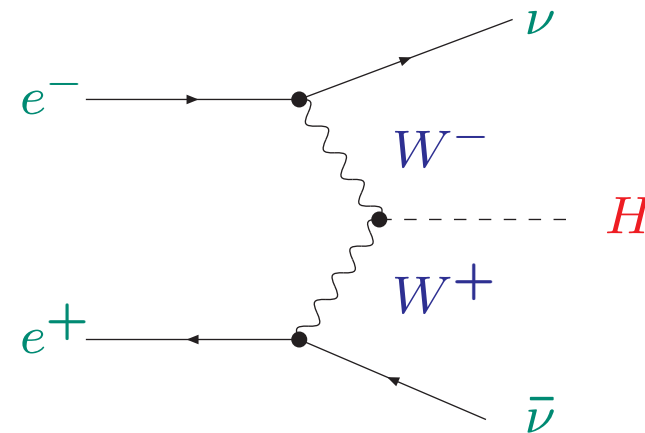
Higgs-strahlung:

$$e^+e^- \rightarrow Z^* \rightarrow ZH$$



weak boson fusion (WBF):

$$e^+e^- \rightarrow \nu\bar{\nu}H$$



$e^+e^- \rightarrow ZH$ :

$$\delta\sigma_{HZ}^{\text{exp}} \sim 0.4\%$$

full one-loop available, corrections of 5-10%

rough estimate:  $\delta\sigma_{HZ}^{\text{theo}} \sim 1\%$  from missing two-loop corrections

Two-loop corrections for  $2 \rightarrow 2$  can in principle be done ...

$\mathcal{O}(\alpha_t\alpha_s)$  corrections: 1.3% [Y. Gong, Z. Li, X. Xu, L. Yang '16]

$\Rightarrow$  theory uncertainties sufficiently small

$\Rightarrow$  full two-loop for  $2 \rightarrow 2$  should be done!

$e^+e^- \rightarrow \nu\bar{\nu}H$ :

small contribution ...

Partial two-loop calculation (with closed fermion loops)  
can in principle be done ...

$\Rightarrow$  theory uncertainties sufficiently small

## Decay width theoretical uncertainties: General recipe:

[LHCHXSWG BR group '15]

### 1. Parametric Uncertainties: $p \pm \Delta p$

- Evaluate partial widths and BRs with  $p$ ,  $p + \Delta p$ ,  $p - \Delta p$  and take the differences w.r.t. central values
- Upper ( $p + \Delta p$ ) and lower ( $p - \Delta p$ ) uncertainties summed in quadrature to obtain the **Combined Parametric Uncertainty**

### 2. Theoretical Uncertainties:

- Calculate uncertainty for partial widths and corresponding BRs for each theoretical uncertainty
- Combine the individual theoretical uncertainties linearly to obtain the **Total Theoretical Uncertainty**

⇒ estimate based on “what is included in the codes”!

### 3. Total Uncertainty:

Linear sum of the Combined Parametric Uncertainty and the Total Theoretical Uncertainties

# Intrinsic uncertainties for decay widths:

[arXiv:1905.03764]

“ILC/FCC-ee” = expected precision on  $g_{Hxx}^2$  (incl. HL-LHC meas.)

Partial width	QCD	electroweak	total	future	FCC-ee/CEPC
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	1.4/1.5%
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	2.6/2.6%
$H \rightarrow \tau^+\tau^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	1.5/1.5%
$H \rightarrow \mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	7.8/7.8%
$H \rightarrow gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$	$\sim 1\%$	2.0/2.0%
$H \rightarrow \gamma\gamma$	$< 0.1\%$	$< 1\%$	$< 1\%$	$< 1\%$	2.6/2.6%
$H \rightarrow Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$	$\sim 1\%$	22/22%
$H \rightarrow WW \rightarrow 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.4\%$	1.0/1.2%
$H \rightarrow ZZ \rightarrow 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.3\%$	0.5/0.5%
$\Gamma_{\text{tot}}$				$\sim 0.3\%$	1.1/1.2%

$\Rightarrow$  non-negligible for  $H \rightarrow WW/ZZ \rightarrow 4f$

## Future parametric uncertainties for decay widths:

decay	fut. intr.	fut. para. $m_q$	para. $\alpha_s$	para. $M_H$	ILC/FCC-ee
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	—	1.4/1.5%
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	—	2.6/2.6%
$H \rightarrow \tau^+\tau^-$	$< 0.1\%$	—	—	—	1.5/1.5%
$H \rightarrow \mu^+\mu^-$	$< 0.1\%$	—	—	—	7.8/7.8%
$H \rightarrow gg$	$\sim 1\%$		0.5%	—	2.0/2.0%
$H \rightarrow \gamma\gamma$	$< 1\%$	—	—	—	2.6/2.6%
$H \rightarrow Z\gamma$	$\sim 1\%$	—	—	$\sim 0.1\%$	22/22%
$H \rightarrow WW$	$\lesssim 0.4\%$	—	—	$\sim 0.1\%$	0.5/0.5%
$H \rightarrow ZZ$	$\lesssim 0.3\%$	—	—	$\sim 0.1\%$	0.4/0.5%
$\Gamma_{\text{tot}}$	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	1.1/1.2%

$\Gamma_{\text{tot}}$  applies “to all” (partial cancelations ...)

$\Rightarrow$  non-negligible in particular for  $H \rightarrow WW/ZZ \rightarrow 4f$  ( $\delta m_b$  optimistic?)

## Future theory uncertainties?

### Intrinsic uncertainties:

$H \rightarrow b\bar{b}, H \rightarrow c\bar{c}$ : higher-order EW corrections ??

$H \rightarrow \tau^+\tau^-, H \rightarrow \mu^+\mu^-$ : higher-order EW corrections ?

$H \rightarrow gg$ : improvement difficult

$H \rightarrow \gamma\gamma$ : already very precise ...

$H \rightarrow Z\gamma$ : EW corrections could help ...

$H \rightarrow WW^{(*)}, H \rightarrow ZZ^{(*)}$ : already very precise, two-loop corrections unclear

$\Rightarrow$  intrinsic uncertainty can/will be sufficiently under control?!

### Parametric uncertainties:

- largely driven by  $\delta m_b \Rightarrow$  improvement unclear (to me)  
lattice community does not seem to agree
- some improvement in  $\alpha_s$  possible



$$\sigma_{Zh} = \left| \text{tree-level} \right|^2 + 2 \operatorname{Re} \left[ \text{tree-level} \cdot \left( \text{loop correction 1} + \text{loop correction 2} \right) \right]$$

$$\delta_{\sigma}^{240} = 100 (2\delta_Z + 0.014\delta_h) \%$$

⇒ sensitivity to  $\lambda_{HHH}$  goes down for higher  $\sqrt{s}$

⇒ percent precision possible on  $\sigma_{ZH}$ ,  $\lambda_{HHH}$

⇒ indirect and model dependent measurement  
(to be included in a global coupling fit - within a model)

⇒  $\mathcal{O}(10\%)$  measurement of  $\lambda_{HHH}$  needed  
to measure  $\sigma_{HZ}$  at the percent level!

⇒ higher  $\sqrt{s}$  needed!

## One word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

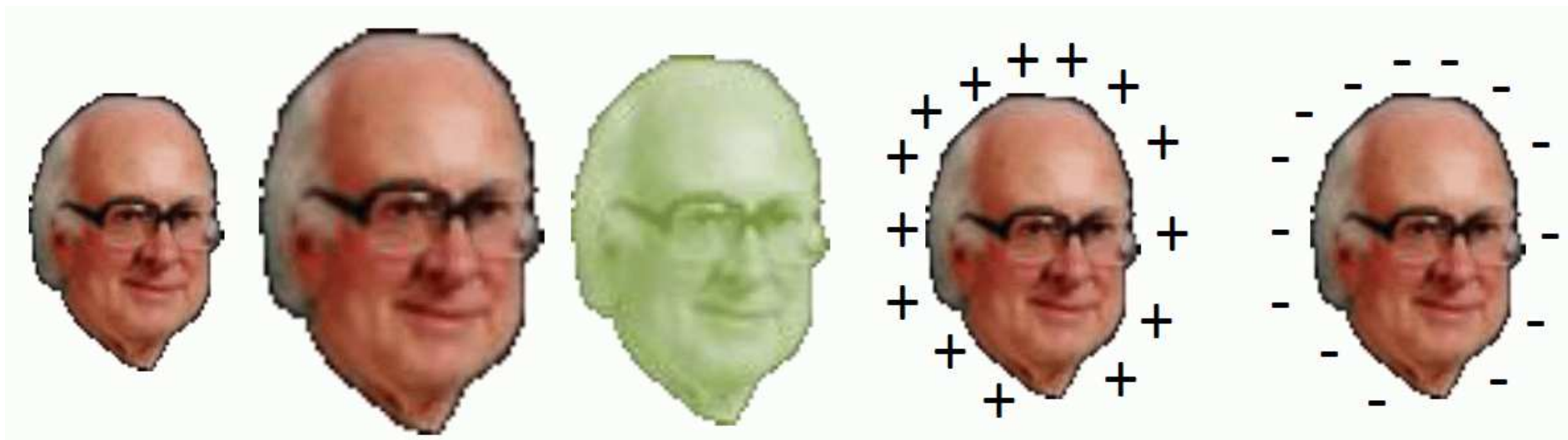
We know that BSM physics must exist! (DM, gravity, ...)

As soon as BSM physics will be discovered, an evaluation of the Higgs predictions in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

## 6. BSM Higgs (the difficult case)



## Required precision for Higgs couplings?

MSSM example:

$$\kappa_V \approx 1 - 0.5\% \left( \frac{400 \text{ GeV}}{M_A} \right)^4$$

$$\kappa_t = \kappa_c \approx 1 - \mathcal{O}(10\%) \left( \frac{400 \text{ GeV}}{M_A} \right)^2 \cot^2 \beta$$

$$\kappa_b = \kappa_\tau \approx 1 + \mathcal{O}(10\%) \left( \frac{400 \text{ GeV}}{M_A} \right)^2$$

Composite Higgs example:

$$\kappa_V \approx 1 - 3\% \left( \frac{1 \text{ TeV}}{f} \right)^2$$

$$\kappa_F \approx 1 - (3 - 9)\% \left( \frac{1 \text{ TeV}}{f} \right)^2$$

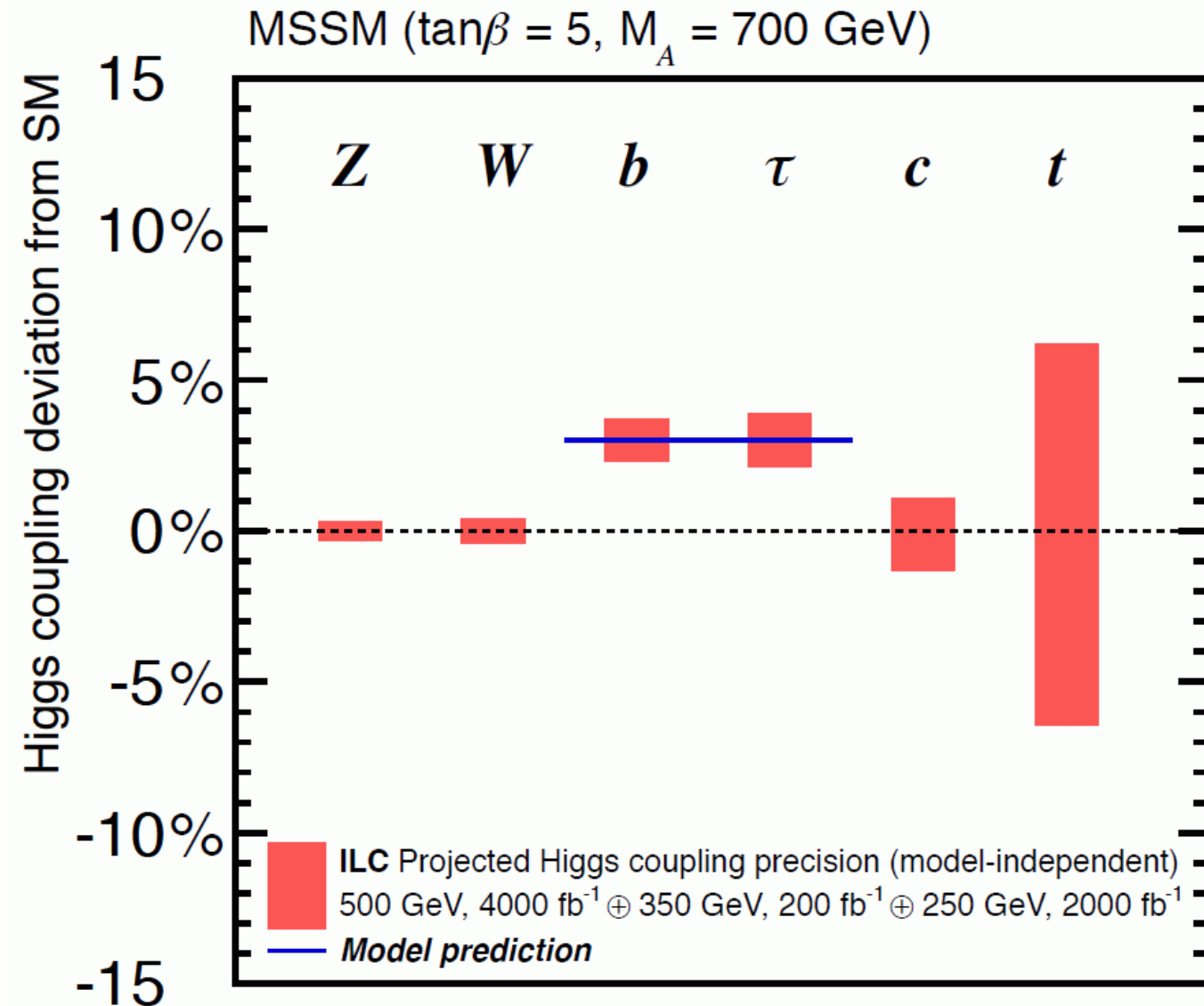
⇒ couplings to bosons in the **per mille** range

⇒ couplings to fermions in the **per cent** range

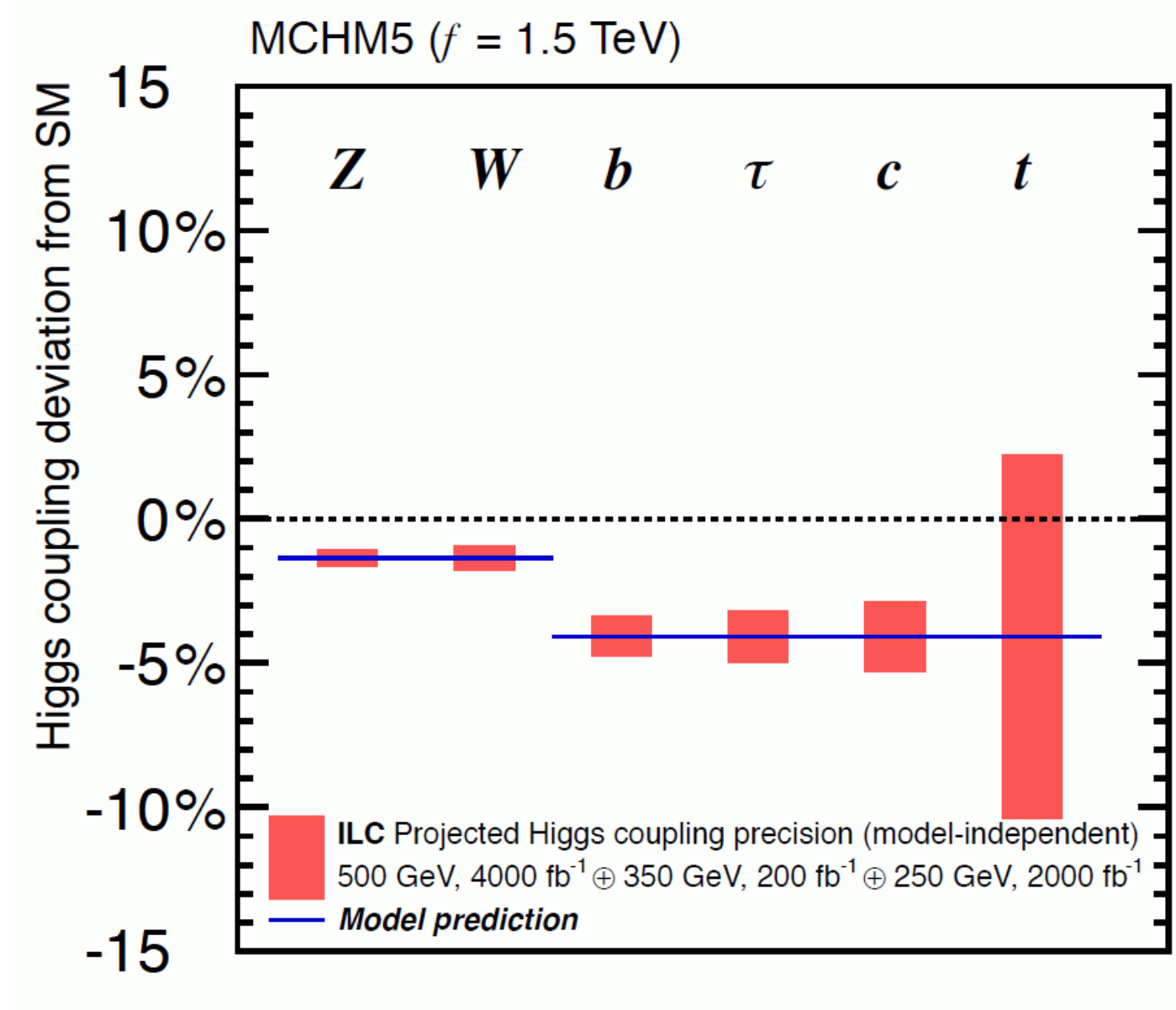
⇒ the more precise the better

⇒ **theory match?**

## ILC precision vs. MSSM prediction:



## ILC precision vs. Composite Higgs prediction:



## Required precision for $\mathcal{CP}$ -admixture?

$$H = \cos \alpha \mathcal{CP}\text{-even} + \sin \alpha \mathcal{CP}\text{-odd}$$

$$\mathcal{A}(X \rightarrow VV) = \frac{1}{v} \left( a_1 m_V^2 \varepsilon_1^* \varepsilon_2^* + a_2 f_{\mu\nu}^{*(1)} f^{*(2),\mu\nu} + a_3 f_{\mu\nu}^{*(1)} \tilde{f}^{*(2),\mu\nu} \right)$$

$$\mathcal{A}(X \rightarrow f \bar{f}) = \frac{m_f}{v} \bar{u}_2 (b_1 + i b_2 \gamma_5) u_1$$

$$f_{\mathcal{CP}} = \frac{|a_3|^2 \sigma_3}{\sum |a_i|^2 \sigma_i}$$

Desired precision:

gauge bosons:  $f_{\mathcal{CP}} \lesssim 10^{-5}$  (loop suppressed)

fermions:  $f_{\mathcal{CP}} \lesssim 10^{-2}$

## Taking the MSSM Higgs production as show case:

⇒ “best case” of “difficult case”!

### Neutral Higgs production:

$$e^+e^- \rightarrow h_i Z, h_i \gamma, h_i h_j, h_i \nu \bar{\nu}, h_i e^+ e^-, h_i t \bar{t}, h_i b \bar{b}, \dots \quad (i, j = 1, 2, 3).$$

Now available in the cMSSM at the full one-loop level:

[S.H., C. Schappacher '15] [F. Arco, S.H., C. Schappacher '18]

$$\sigma(e^+e^- \rightarrow h_i h_j)$$

$$\sigma(e^+e^- \rightarrow h_i Z)$$

$$\sigma(e^+e^- \rightarrow h_i \gamma)$$

⇒ no dedicated two-loop corrections available yet

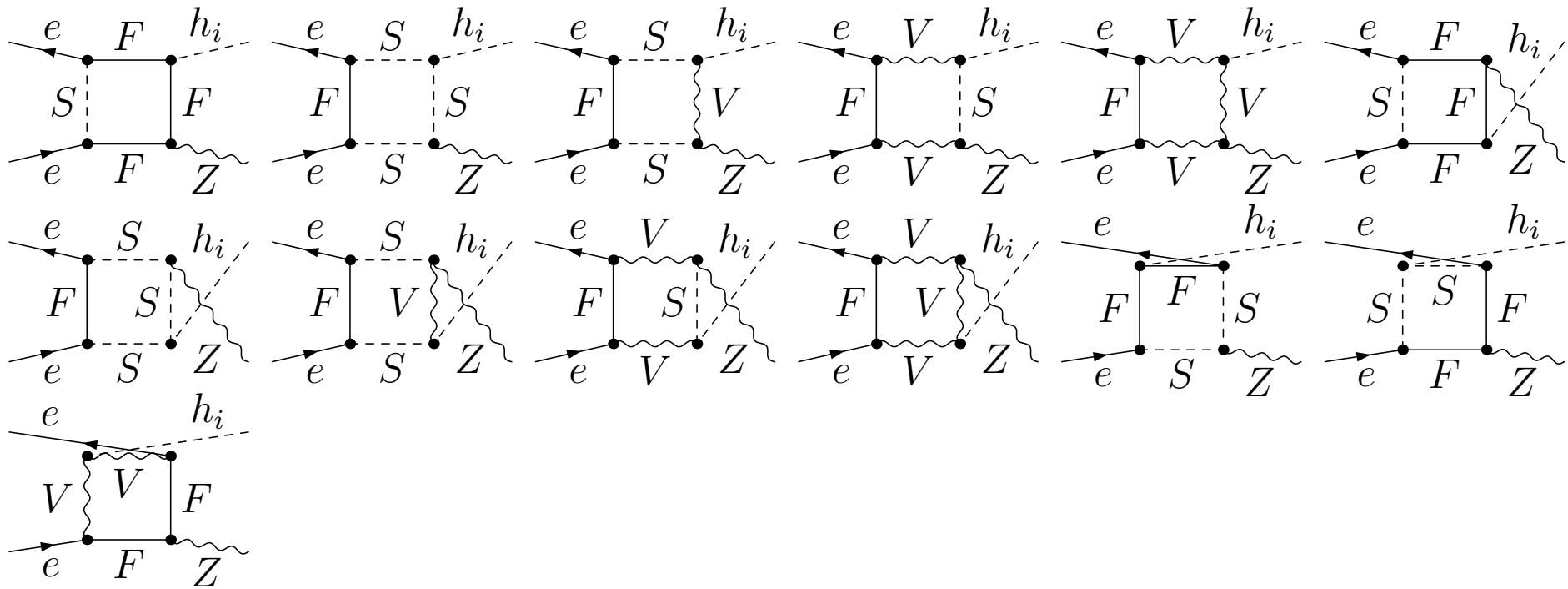
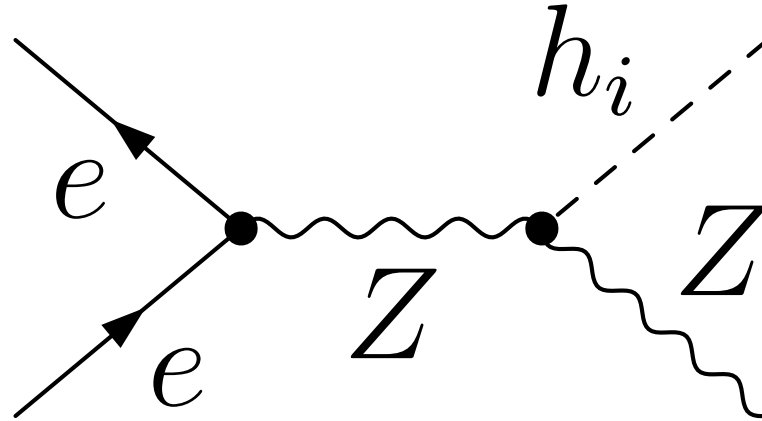
⇒ as in SM full two-loop would be needed (possible ...)

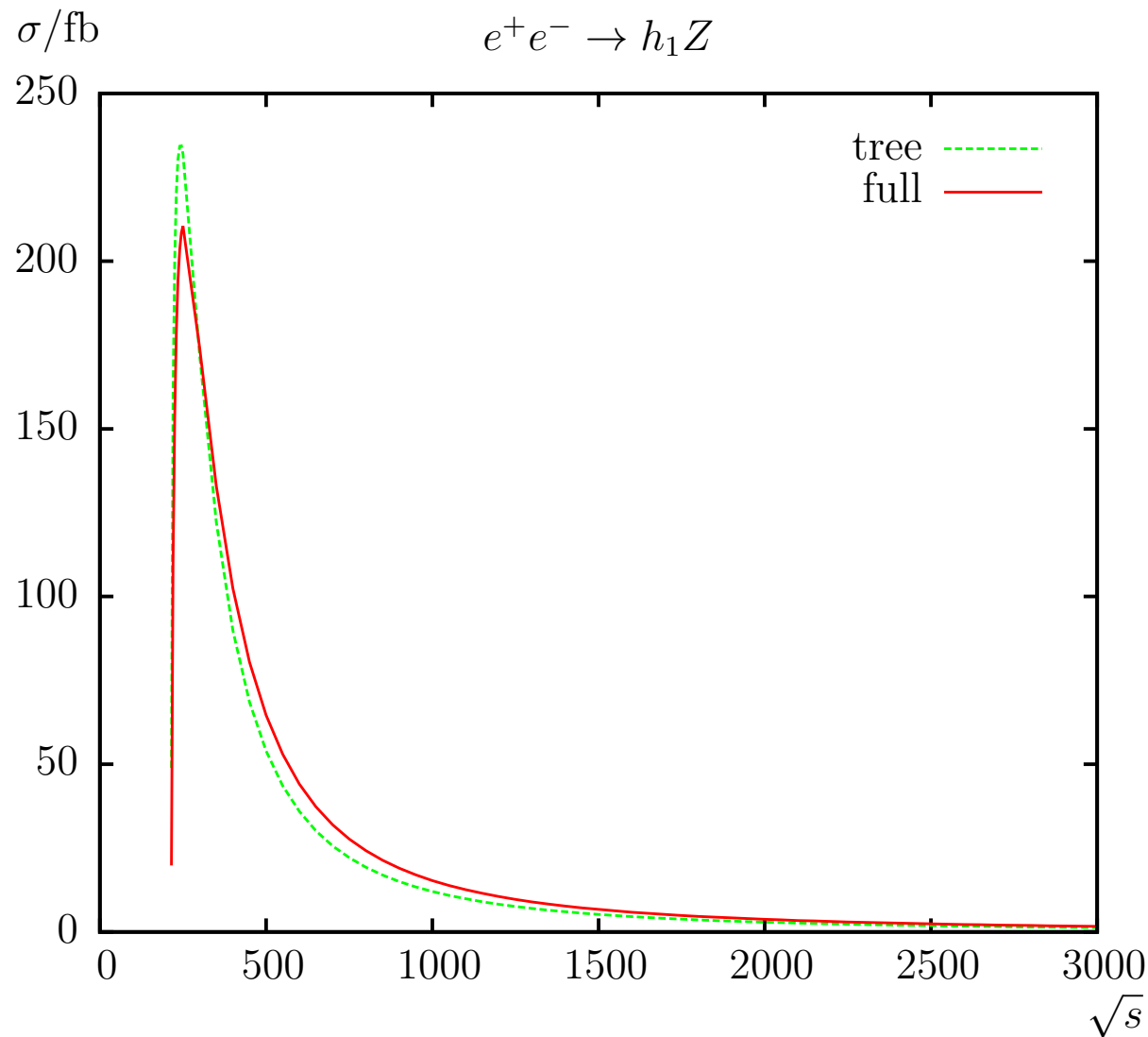
Remember: more neutral Higgs production channels

⊕ charged Higgs production channels!

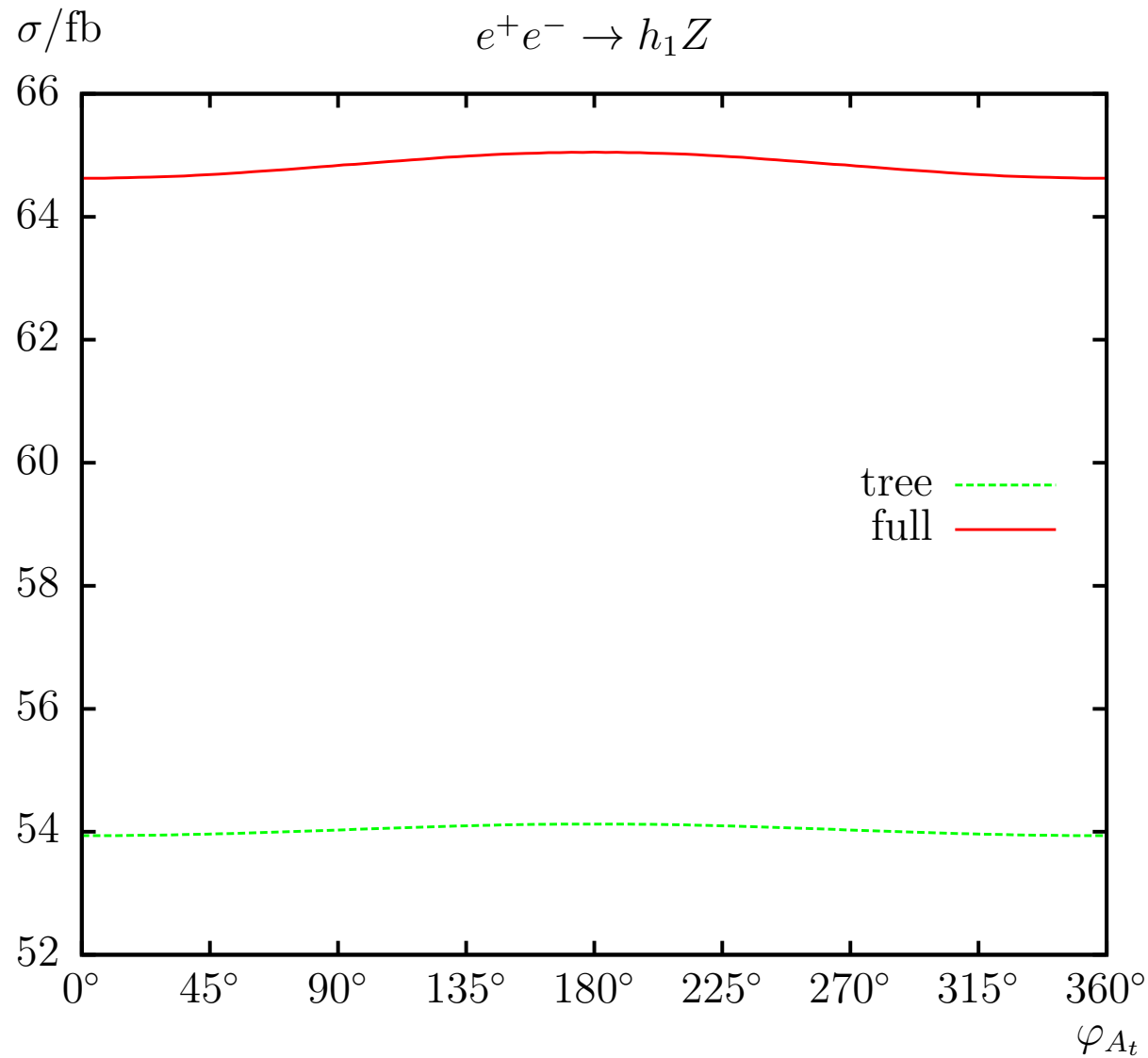


$$\underline{e^+e^- \rightarrow h_i Z:}$$





$\Rightarrow$  loop corrections crucial       $\Rightarrow$  two-loop required



⇒ complex parameters have per-cent effects, has to be included!

## Most complete implementation in cMSSM: FeynHiggs

### Evaluation of all MSSM Higgs boson masses and mixing angles

- $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}$  ,  $\alpha_{\text{eff}}$ ,  $Z_{ij}$ ,  $U_{ij}$ , ...

### Evaluation of all neutral MSSM Higgs boson decay channels (so far)

- total decay width  $\Gamma_{\text{tot}}$
- $\text{BR}(h_i \rightarrow f\bar{f})$ : decay to SM fermions: full 1L, running  $m_q$  at 3L,  $Z_{ij}$
- $\text{BR}(h_i \rightarrow Z^{(*)}Z^{(*)}, W^{(*)}W^{(*)})$ : decay to massive SM gauge bosons: Prophecy4f  $\oplus$  coupling factors,  $U_{ij}$
- $\text{BR}(h_i \rightarrow \gamma\gamma, gg)$ : decay to massless SM gauge bosons: NLO QCD,  $gg$ : NNLO, NNLL from SM,  $U_{ij}$
- $\text{BR}(h_i \rightarrow h_j Z^{(*)}, h_j h_k)$ : decay to gauge and Higgs bosons:  $h_j Z^{(*)}$ :  $U_{ij}$ ,  $h_j h_k$ : full 1L, log-resum,  $Z_{ij}$
- $\text{BR}(h_i \rightarrow \tilde{f}_i \tilde{f}_j)$ : decay to sfermions:  $U_{ij}$
- $\text{BR}(h_i \rightarrow \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp, \tilde{\chi}_i^0 \tilde{\chi}_j^0)$ : decay to charginos, neutralinos:  $U_{ij}$

## Overall (N)MSSM Higgs decay intrinsic uncertainty estimates

[F. Domingo, S.H., S. Passehr, G. Weiglein '18]

- $h_i \rightarrow q\bar{q}$ : SM-like: SM NNLO QCD, EW NNLO, SUSY 2L:  $\sim 5\%$   
heavy: as SM-like, Sudakov logs:  $\sim 5 - 10\%$
- $h_i \rightarrow \ell\bar{\ell}$ : SM-like:  $\lesssim 1\%$   
heavy: Sudakov logs for very heavy Higgses  $\lesssim 10\%$
- $h_i \rightarrow WW^{(*)}, ZZ^{(*)}$ : SM-like:  $\lesssim 1\%$   
heavy: missing 2L (very small width):  $\lesssim 50\%$
- $h_i \rightarrow \gamma\gamma, gg, \gamma Z$ :  $\gamma\gamma$ : NNLO QCD, EW:  $\lesssim 4\%$   
 $gg$ : NNLO QCD, EW:  $\lesssim 4\%$   
 $\gamma Z$ : NLO:  $\sim 5\%$
- $h_i \rightarrow \text{SUSY SUSY}$ : [S.H., C. Schappacher '14-'16]  
1L effects  $10 - 20\%$ , 2L?
- all decays:  $U_{ij}, Z_{ij}$ : few %, effects close to threshold?

$\Rightarrow$  approaching CEPC precision for SM-like Higgs  
(not for heavy Higgses yet)

## 7. Conclusions

- High anticipated experimental precision for Higgs/EWPO at future  $e^+e^-$  colliders
- Crucial: theory uncertainties: intrinsic and parametric

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

- We give (realistic/optimistic) estimates for future intrinsic and parametric uncertainties
- EWPO: intrinsic unc. larger than anticipated experimental unc.  
parametric unc. often larger than experimental uncertainties  
 $\Rightarrow$  particularly true for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$
- SM Higgs: cross section can be under control with full  $2 \rightarrow 2$  calc.  
intrinsic unc. can be relevant for  $H \rightarrow WW/ZZ \rightarrow 4f$   
parametric unc. can be relevant, in particular for  $H \rightarrow WW/ZZ \rightarrow 4f$
- BSM Higgs: deviations in per-cent range expected  
 $\Rightarrow$  MSSM is “best case” of “difficult case”!  
cross sections can be under control with full  $2 \rightarrow 2$  calc.  
intrinsic unc. approaching  $e^+e^-$  precision for SM-like Higgs  
parametric unc. at least as large as in SM
- Uncertainties should be taken into account by experimental analyses!

A still from a Star Wars movie showing a man in a grey uniform looking up at Darth Vader. The scene is set in a dark, industrial environment with blue lighting. The text "Further Questions?" is overlaid in white.

Further Questions?